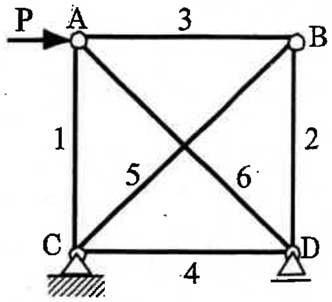
構造力学Ⅲ/Ⅱ　期末演習（令和元年度）

2020.1.31

問題1　右図の正方形単純支持のトラス構造がA点に水平方向集中力Pを受けた。各材の伸縮剛さはEAとし、1、2、3、4部材の長さはaとする。斜材1の部材力を求めよ。【2点】

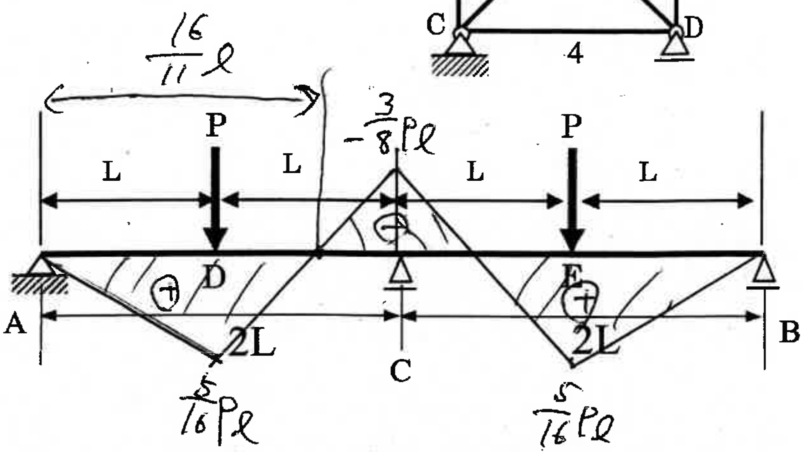


$$N_1 = \frac{P}{2}$$

問題2. 右図の2径間連続梁について以下の設問に答えよ．但し，曲げ剛性EI一定とし，曲げ剪断歪みエネルギーは無視出来るものとする．

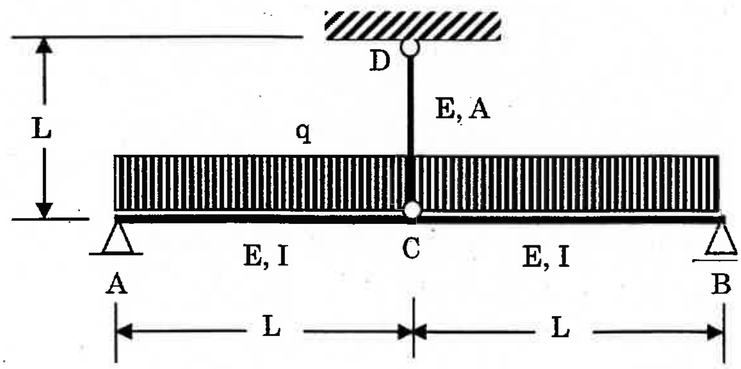
(1) C点の支点反力を求めよ．　【2点】

(2) 曲げモーメント図を求めよ．【2点】



$$V_c = \frac{11}{8}P$$

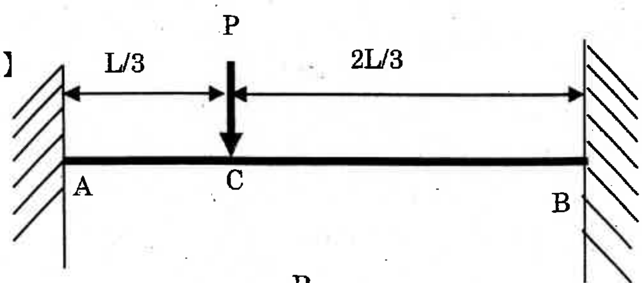
問題3. 右図に示す等分布荷重qを受ける単純支持梁の中点Cを天井から引張材CDで吊られた構造物がある。C点のたわみを求めよ。但し、梁曲げ剛性はEI、引張材の伸び剛性EAとし、曲げ剪断歪みエネルギーは無視出来るものとする。【1点】



$$\Delta_c = \frac{5ql^4}{4\left(EA + \frac{6EI}{l^2}\right)}$$

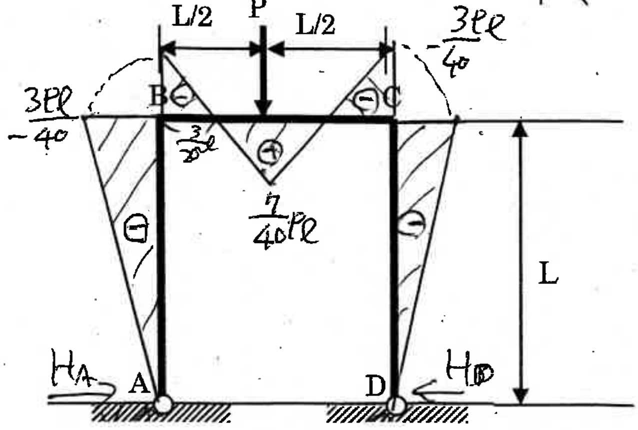
問題4. 右図に示す両端固定支持に集中荷重が作用する構造系に対して，下式のたわみ角公式（材端モーメント式）の中間荷重項 C_{AB} を求めよ。【2点】

$$M_{AB} = 2EI/L\,(2\theta_A + \theta_B - 3R) + C_{AB}$$



$$C_{AB} = -\frac{Pab^2}{l^2} = -\frac{4}{27}Pl$$

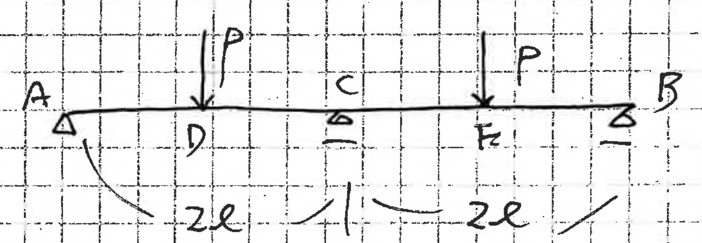
問題5. 右図に示すA点とD点がピン支持されているラーメン構造に、図のように集中荷重Pが作用しているときの曲げモーメント図を求めよ。但し，曲げ剛性EI一定とし、曲げ剪断歪みエネルギーは無視出来るものとする。【1点】



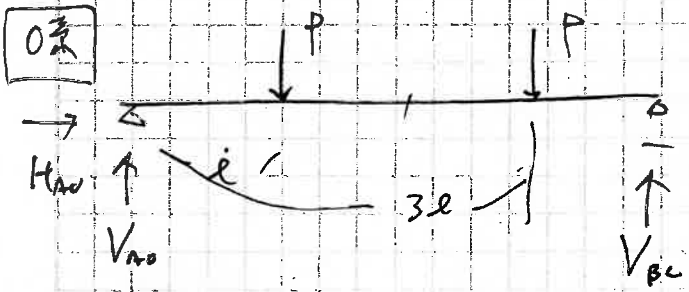
$$V_A = V_D = \frac{P}{2}$$

$$H_A = H_D = \frac{3}{40}P$$

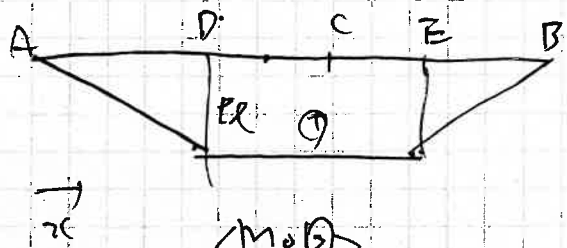
問題3



0系



$$\Sigma V = V_{A0} + V_{B0} - P - P = 0$$
$$V_{A0} + V_{B0} - 2P = 0$$
$$\Sigma M_A = Pl + 3Pl - 4V_{B0}l = 0$$
$$V_{B0} = P = V_{A0}$$



〈M_0図〉

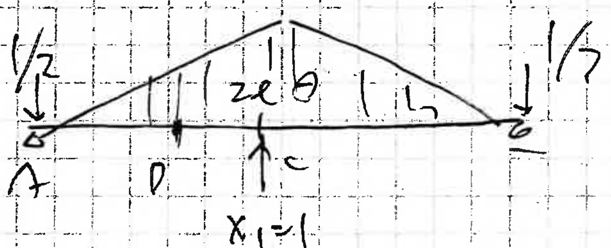
A〜D間

$$M_{x0} = Px$$

D〜C間

$$M_{x0} = V_{A0}x - P\times(x-l)$$
$$= Px - Px + Pl = Pl$$

1系



〈M_{x1}図〉

A〜D間

$$M_{x1} = -x/2$$

D〜C間

$$M_{x1} = -x/2$$

① 変形適合条件

$$\Delta_c = \Delta_{c0} + X_1\Delta_{c1} = 0$$

$$\Delta_{c0} = \frac{2}{EI}\left[\int_0^l \left\{Px\times\left(-\frac{x}{2}\right)\right\}dx + \int_l^{2l}\left\{Pl\times\left(-\frac{x}{2}\right)\right\}dx\right]$$

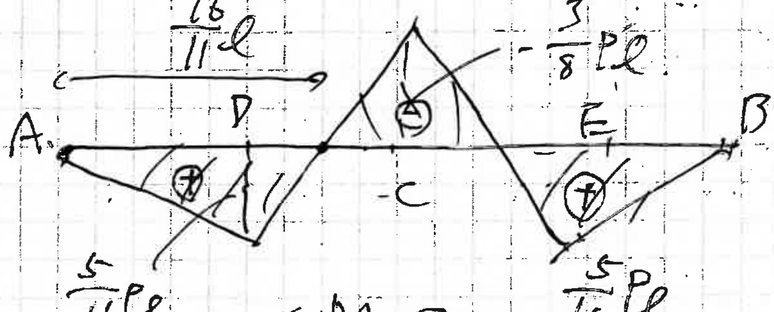
$$= \frac{2}{EI}\left\{\int_0^l -\frac{P}{2}x^2dx + \int_l^{2l} -\frac{1}{2}Plx\,dx\right\} = \frac{2}{EI}\left\{\left[-\frac{P}{6}x^3\right]_0^l + \left[-\frac{1}{4}Plx^2\right]_l^{2l}\right\}$$

$$= \frac{2}{EI}\left\{-\frac{1}{6}Pl^3 + \left(-Pl^3 + \frac{1}{4}Pl^3\right)\right\} = \frac{2}{EI}\left(-\frac{Pl^3}{6} - Pl^3 + \frac{Pl^3}{4}\right) = -\frac{11Pl^3}{6EI}$$

$$\Delta_{11} = \frac{2}{EI}\left\{\int_0^{2l}\left(\frac{x^2}{4}\right)dx\right\} = \frac{2}{EI}\left[\frac{x^3}{12}\right]_0^{2l} = \frac{l^3}{EI}\times\frac{2\times 8}{12} = \frac{l^3}{EI}\times\frac{4}{3}$$

$$= \frac{4l^3}{3EI}$$

$$X_1 = -\frac{\Delta_{10}}{\Delta_{11}} = \frac{11Pl^3}{6EI}\times\frac{3EI}{4l^3} = \frac{11}{8}P = V_C$$



<Mx図>

A~D間

$$M_x = Px + \frac{11}{8}P\times\left(-\frac{x}{2}\right) = Px - \frac{11}{16}Px = \frac{5}{16}Px$$

D~C間

$$M_x = Pl + \frac{11}{8}P\times\left(-\frac{x}{2}\right) = Pl - \frac{11}{16}Px$$

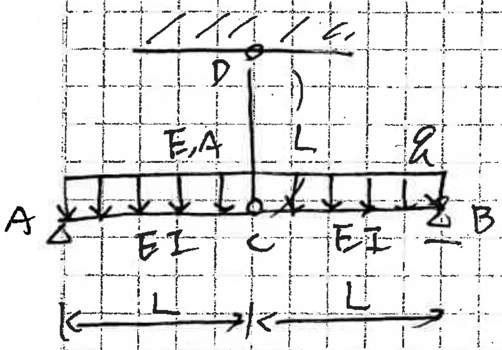
$x = 2l$ のとき, $M_x = Pl - \frac{22}{16}Pl = -\frac{6}{16}Pl = -\frac{3}{8}Pl$

$M_x = Pl - \frac{11}{16}Px = 0$ より,

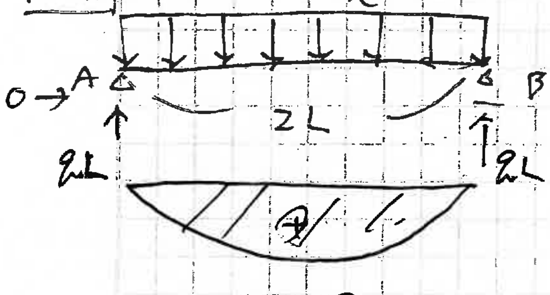
$$\frac{11}{16}Px = Pl$$

$$x = \cancel{P}l\times\frac{16}{11\cancel{P}} = \frac{16}{11}l$$

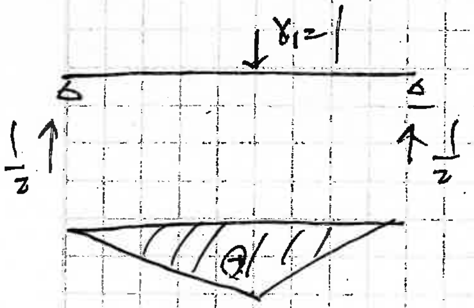
問題3



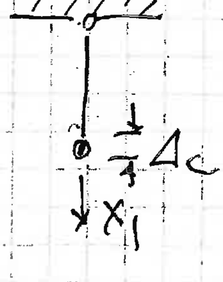
$\boxed{0系}$



〈M_0図〉



〈M_1図〉



〈A〜C間〉

$$\begin{cases} M_{0x} = q l x - \frac{1}{2} q x^2 \\ M_{1x} = \frac{1}{2} x \end{cases}$$

$$EI\Delta_{10} = 2 \times \int_0^l \left(\frac{1}{2} q l x^2 - \frac{q}{4} x^3 \right) dx$$

$$= \frac{5}{24} q l^4$$

$$\therefore \Delta_{10} = \frac{5 q l^4}{24 EI}$$

$$EI\delta_{11} = 2 \times \int_0^l \frac{1}{4} x^2 dx = \frac{l^3}{6}$$

$$\therefore \delta_{11} = \frac{l^3}{6EI}$$

$$\Delta_c = \frac{X_1 l}{EA}$$

変形適合条件式 $\Delta = \Delta_{10} + \delta_{11} X_1 + \Delta_c = 0$

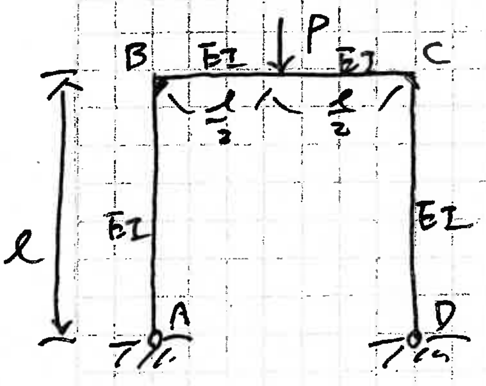
$$\frac{5 q l^4}{24 EI} + \frac{l^3}{6EI} X_1 + \frac{l}{EA} X_1 = 0$$

$$X_1 = \frac{5 q l^3}{4\left(1 + \frac{6EI}{EA l^2}\right)}$$

$$\therefore \Delta_c = \frac{l}{EA} \times \frac{5 q l^3}{4\left(1 + \frac{6EI}{EA l^2}\right)}$$

$$= \frac{5 q l^4}{4\left(EA + \frac{6EI}{l^2}\right)} \;//$$

問題5. 2ヒンジラーメン／荷重対称／構造対称



EI一定，剛比 $k=1$

○たわみ角法で解く

荷重対称，構造対称より 部材角はゼロ

A点とD点はヒンジなので，$M_A = M_D = 0$

実用材端モーメント

$M_{ab} = 2\varphi_a + \varphi_b = 0 \rightarrow \varphi_b = -2\varphi_a$ ── ①

$M_{ba} = 2\varphi_b + \varphi_a = -4\varphi_a + \varphi_a = -3\varphi_a$ ── ②

$M_{dc} = 2\varphi_d + \varphi_c = 0 \rightarrow \varphi_c = -2\varphi_d$ ── ③

$M_{bc} = 2\varphi_b + \varphi_c + C_{bc} = -4\varphi_a - 2\varphi_d + C_{bc}$ ── ④

$M_{cb} = 2\varphi_c + \varphi_b + C_{cb} = -4\varphi_d - 2\varphi_a + C_{cb}$ ── ⑤

$M_{cd} = 2\varphi_c + \varphi_d = -4\varphi_d + \varphi_d = -3\varphi_d$ ── ⑥

節点方程式

・$M_{ba} + M_{bc} = 0$ より，$-3\varphi_a - 4\varphi_a - 2\varphi_d + C_{bc} = 0$

$-7\varphi_a - 2\varphi_d + C_{bc} = 0$ ── ⑦

・$M_{cb} + M_{cd} = 0$ より $-4\varphi_d - 2\varphi_a + C_{cb} - 3\varphi_d = 0$

$-7\varphi_d - 2\varphi_a + C_{cb} = 0$ ── ⑧

⑦より，$\varphi_d = -\frac{7}{2}\varphi_a + \frac{1}{2}C_{bc}$ ── ⑨

⑨を⑧へ代入

$\frac{49}{2}\varphi_a - \frac{7}{2}C_{bc} - 2\varphi_a + C_{cb} = 0$

$\left(C_{bc} = -\frac{Pl}{8}, \quad C_{cb} = \frac{Pl}{8} \right)$

$49\varphi_a - 4\varphi_a = 7C_{bc} - 2C_{cb}$

$45\varphi_a = 7\times\left(-\frac{Pl}{8}\right) - 2\times\frac{Pl}{8}$

$$\varphi_a = \frac{1}{45} \times \frac{-7-2}{8} Pl = \frac{-9}{45\times 8} Pl = -\frac{1}{40} Pl$$

$$\varphi_d = -\frac{7}{2} \times \left(-\frac{1}{40} Pl\right) + \frac{1}{2} \times \left(-\frac{1}{8} Pl\right)$$

$$= \frac{7}{80} Pl - \frac{1}{16} Pl = \frac{7-5}{80} Pl = \frac{1}{40} Pl$$

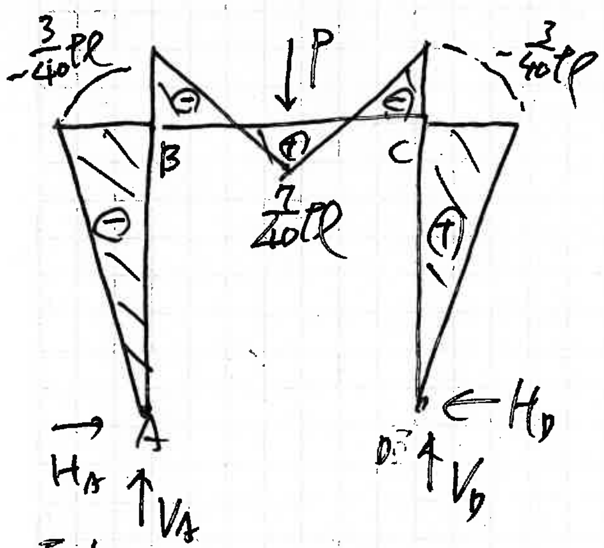
$$\varphi_b = -2 \times \left(-\frac{1}{40} Pl\right) = \frac{1}{20} Pl$$

$$\varphi_c = -2 \times \left(\frac{1}{40} Pl\right) = -\frac{1}{20} Pl$$

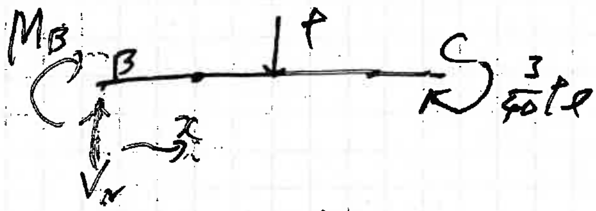
$$M_A = M_D = 0$$

$$M_B = -M_{ba} = 3\varphi_a = 3 \times \left(-\frac{1}{40} Pl\right) = -\frac{3}{40} Pl$$

$$M_C = M_{cd} = -3\times\varphi_d = -3 \times \frac{1}{40} Pl = -\frac{3}{40} Pl$$



B～C間



$$M_x = M_B + V_A x$$

$$= -\frac{Pl}{40} + \frac{P}{2} x$$

$x = \frac{l}{2}$ のとき

$$M_x = -\frac{3Pl}{40} + \frac{Pl}{4} = \frac{-3+10}{40} Pl$$

$$= \frac{7}{40} Pl$$

反力

$$\Sigma H = H_A - H_D = 0$$

$$\Sigma V = V_A + V_D - P = 0$$

$$\Sigma M_A = +\frac{P}{2} l - V_D l = 0$$

$$V_D = \frac{1}{2} P = V_A$$

層方程式より

$$Q_{ab} = -\frac{1}{l}(M_{ab} + M_{ba}) = -\frac{1}{l} \times \left(-\frac{3}{40} Pl\right) = \frac{3}{40} P = H_A = H_D$$