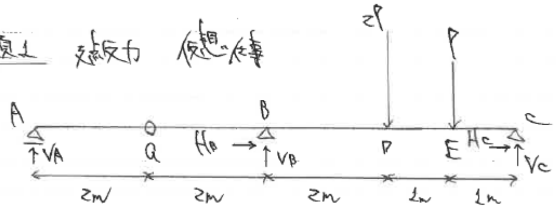
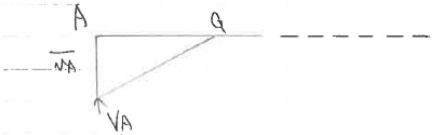
問題1　解答

問題1　支点反力　仮想仕事



$H_B = H_C = 0$

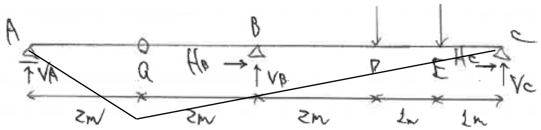
i)　A点の拘束解除

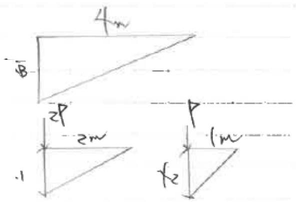


$V_A \cdot (-\bar{\delta}_A) = 0$

$V_A = 0$

ii)　B点の拘束解除





$\bar{\delta}_B : x_1 = 4 : 2$　　$x_1 = \frac{1}{2}\bar{\delta}_B$

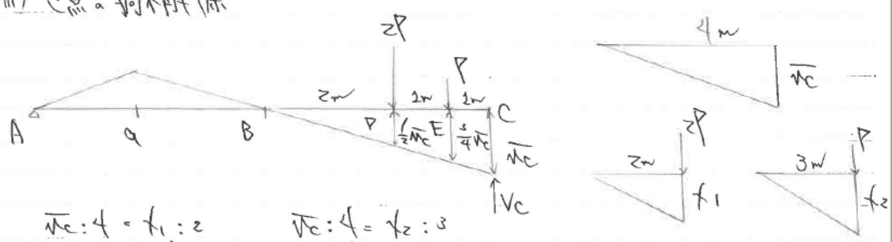
$\bar{\delta}_B : x_2 = 4 : 1$　　$x_2 = \frac{1}{4}\bar{\delta}_B$

$V_B \cdot (-\bar{\delta}_B) + 2P \cdot \frac{1}{2}\cancel{\bar{\delta}_B} + P \cdot \frac{1}{4}\cancel{\bar{\delta}_B} = 0$

$-V_B + P + \frac{P}{4} = 0$

$V_B = \frac{5}{4}P$

iii)　C点の拘束解除



$\bar{\delta}_C : 4 = x_1 : 2$　　$x_1 = \frac{1}{2}\bar{\delta}_C$

$\bar{\delta}_C : 4 = x_2 : 3$　　$x_2 = \frac{3}{4}\bar{\delta}_C$

$V_C \cdot (-\cancel{\bar{\delta}_C}) + 2P \cdot \frac{1}{2}\cancel{\bar{\delta}_C} + P \cdot \frac{3}{4}\cancel{\bar{\delta}_C} = 0$

$-V_C = -\frac{7}{4}P$

$V_C = \frac{7}{4}P$

$V_A = 0$ kN

$V_B = \frac{5}{4}P$ kN

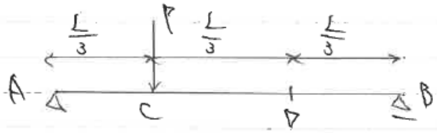
$V_C = \frac{7}{4}P$ kN

$H_B = 0$ kN

$H_C = 0$ kN

問題2　解答

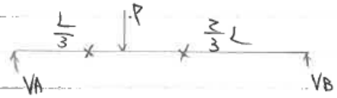
問題2：曲げ剛性 EI = 一定　曲げせん断ひずみエネルギー無視



$W = U$

(1) 仕事エネルギー法　Δ_c

(実系)



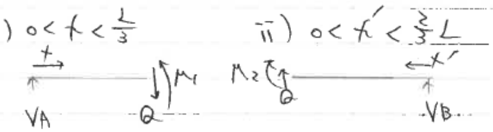
$\Sigma V = V_A + V_B - P = 0$

$\Sigma M_{(A)} = P \cdot \frac{L}{3} - V_B \cdot L = 0$

$V_B = \frac{1}{3}P$

$V_A = \frac{2}{3}P$

i) $0 < x < \frac{L}{3}$　　ii) $0 < x' < \frac{2}{3}L$

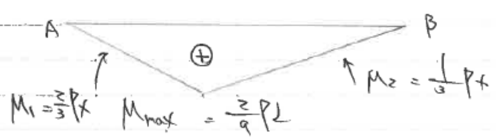


$V_A x - M_1 = 0$

$M_1 = \frac{2}{3}Px$

$M_2 - V_B x' = 0$

$M_2 = \frac{1}{3}Px$



$W = U$

$$\begin{cases} W = \frac{1}{2}P \cdot \Delta_c \\ U = \int_0^L \frac{M_x^2}{2EI}dx \end{cases}$$

$$U = \int_0^L \frac{M_x^2}{2EI}dx$$

$$= \int_0^{\frac{L}{3}} \frac{1}{2EI} \times \left(\frac{2}{3}Px\right)^2 dx + \int_0^{\frac{2}{3}L} \frac{1}{2EI} \times \left(\frac{1}{3}Px\right)^2 dx$$

$$U = \frac{2}{243EI}P^2L^3$$

$W = U$ より、

$$\frac{1}{2}\cancel{P} \cdot \Delta_c = \frac{2}{243EI}P^{\cancel{2}}L^3$$

$$\Delta_c = \frac{4}{243EI}PL^3$$

$$\Delta_c = \frac{4PL^3}{243EI}$$

(2) ポテンシャルエネルギー法　Δ_c

$U = \frac{2}{243EI}P^2L^3$　　$\frac{\partial U}{\partial P}$

$\Pi = V + U$ より、

$\Pi = -P \cdot \Delta_c + \frac{2}{243EI}P^2L^3$

$\frac{\partial \Pi}{\partial P} = -\Delta_c + \frac{4}{243EI}PL^3$

$\partial \Pi = \left(-\Delta_c + \frac{4}{243EI}PL^3\right)\partial P$

$\partial P \neq 0$ より、　$-\Delta_c + \frac{4}{243EI}PL^3 = 0$

$$\Delta_c = \frac{4PL^3}{243EI}$$

問題2　解答

問題2

(3) 仮想仕事法 Δ_D

仮想仕事法を用いる時は積分区間を揃える

実系において、

$0<x<\frac{L}{3} \Rightarrow M_1=\frac{2}{3}Px$

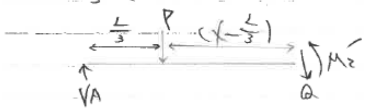
$0<x'<\frac{2}{3}L \Rightarrow M_2=\frac{1}{3}Px$ より、

↓

$\frac{L}{3}<x<L$ で再算出

(実系) $0<x<\frac{L}{3} \Rightarrow \frac{2}{3}Px$

$\frac{L}{3}<x<L$ のとき、



$\Sigma M: V_A x - P(x-\frac{L}{3}) - M_2' = 0$

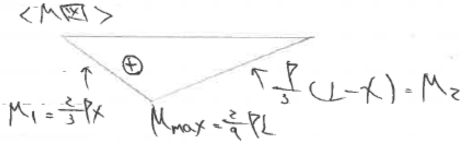
$\frac{2}{3}Px - Px + \frac{P}{3}L = M_2'$

$M_2' = \frac{P}{3}L - \frac{P}{3}x$

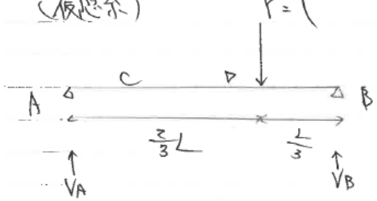
$M_2 = \frac{P}{3}(L-x)$

$(M_2' = M_2)$

<M図>



(仮想系)　$\bar{P}=1$　$\bar{P}=1$ を作用

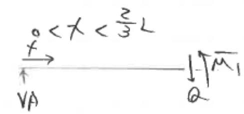


$\Sigma V: V_A+V_B-1=0$

$\Sigma M: \frac{2}{3}L\cdot 1 - V_B L = 0$

$V_B=\frac{2}{3}$　$V_A=\frac{1}{3}$

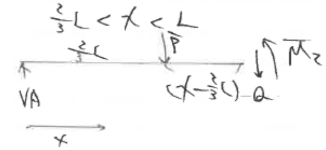
$0<x<\frac{2}{3}L$



$\Sigma M: V_A x - \bar{M}_1 = 0$

$\bar{M}_1=\frac{1}{3}x$

$\frac{2}{3}L<x<L$

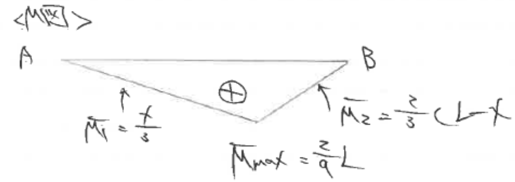


$\Sigma M: V_A x - 1(x-\frac{2}{3}L) - \bar{M}_2=0$

$\frac{x}{3} - x + \frac{2}{3}L = \bar{M}_2$

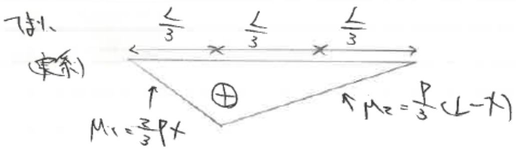
$\bar{M}_2 = \frac{2}{3}(L-x)$

<M図>



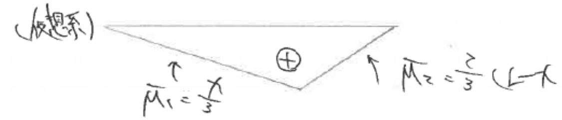
つまり、

(実系)



$\Delta_D=\int_0^{l} \frac{M\bar{M}}{EI}dx$

(仮想系)



よって、 $\Delta_D = \underbrace{\int_0^{\frac{L}{3}}\frac{(\frac{2}{3}Px)(\frac{x}{3})}{EI}dx}_{①} + \underbrace{\int_{\frac{L}{3}}^{\frac{2}{3}L}\frac{\frac{P}{3}(L-x)(\frac{x}{3})}{EI}dx}_{②} + \underbrace{\int_{\frac{2}{3}L}^{L}\frac{\frac{P}{3}(L-x)\cdot\frac{2}{3}(L-x)}{EI}dx}_{③}$

↓

それぞれを計算

①、②、③より、

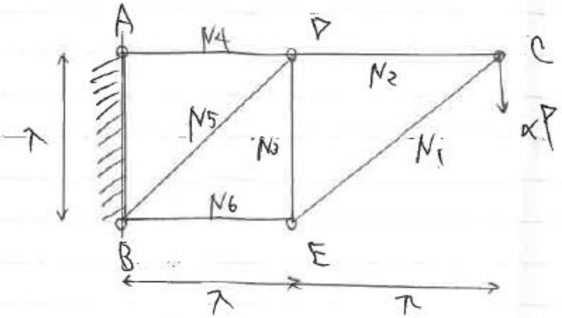
$\Delta_D = \frac{2PL^3}{729EI}+\frac{13PL^3}{1458EI}+\frac{2PL^3}{729EI}$

$=\frac{4+13+4}{1458EI}PL^3 = \frac{21}{1458EI}PL^3 = \frac{7}{486EI}PL^3$

$\Delta_D = \frac{7}{486EI}PL^3$ //

問題3　解答

問題3　Δ_C　伸縮剛性 = EA



i	L_i	N_i	N_i^2	$N_i^2 L$
1	$\sqrt{2}\lambda$	$-\alpha\sqrt{2}P$	$2\alpha^2P^2$	$2\sqrt{2}\alpha^2P^2\lambda$
2	λ	αP	α^2P^2	$\alpha^2P^2\lambda$
3	λ	αP	α^2P^2	$\alpha^2P^2\lambda$
4	λ	$2\alpha P$	$4\alpha^2P^2$	$4\alpha^2P^2\lambda$
5	$\sqrt{2}\lambda$	$-\alpha\sqrt{2}P$	$2\alpha^2P^2$	$2\sqrt{2}\alpha^2P^2\lambda$
6	λ	$-\alpha P$	α^2P^2	$\alpha^2P^2\lambda$

$$U = \Sigma \frac{N^2L}{2EA}$$

$$= \frac{1}{2EA}\left(2\sqrt{2}+1+1+4+2\sqrt{2}+1\right)\alpha^2P^2\lambda$$

$$= \frac{\alpha^2P^2\lambda}{EA}\left(7+4\sqrt{2}\right)$$

$$W = \frac{1}{2}\alpha P \Delta_C$$

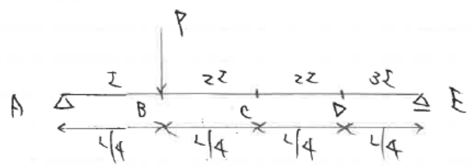
$W = U$ より、 $\frac{1}{2}\alpha P \Delta_C = \frac{\alpha^2P^2\lambda}{2EA}\left(7+4\sqrt{2}\right)$

$$\Delta_C = \frac{\alpha P\lambda}{EA}\left(7+4\sqrt{2}\right)$$

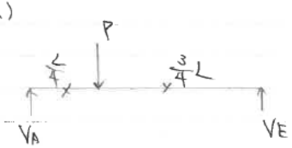
問題4　解答

問題4　鉛直下方変位 Δ_C　　I, $2I$, $3I$　　E は一定

曲げ剪断ひずみエネルギー は無視



(実系)



$\Sigma V: V_A + V_E - P = 0$

$\Sigma M_{(A)}: P \cdot \frac{L}{4} - V_E L = 0$

$V_E = \frac{P}{4}$

$V_A + \frac{P}{4} - P = 0$

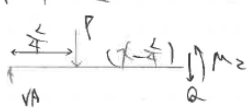
$V_A = \frac{3}{4}P$

$0 < x < \frac{L}{4}$

$\Sigma M: V_A x - M_1 = 0$

$M_1 = \frac{3}{4}Px$

$\frac{L}{4} < x < L$

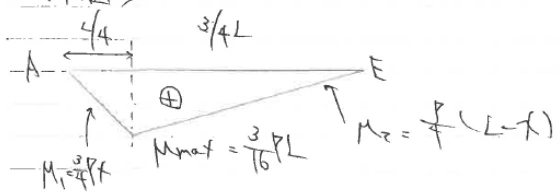


$\Sigma M: V_A x - P(x - \frac{L}{4}) - M_2 = 0$

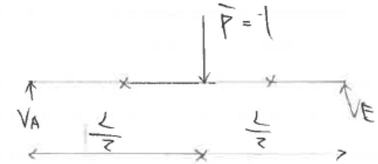
$\frac{3}{4}Px - Px + \frac{P}{4}L - M_2 = 0$

$M_2 = \frac{PL}{4} - \frac{Px}{4} = \frac{P}{4}(L - x)$

< M図 >



(仮想系)　$\bar{P} = 1$ を点Cに及ぼす



$\Sigma V: V_A + V_E - 1 = 0$

$\Sigma M: 1 \cdot L/2 - V_E L = 0$

$V_E = \frac{1}{2}$

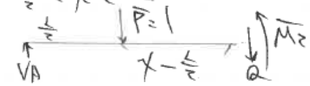
$V_A = \frac{1}{2}$

$0 < x < \frac{L}{2}$

$\Sigma M: V_A x - \bar{M}_1 = 0$

$\bar{M}_1 = \frac{x}{2}$

$\frac{L}{2} < x < L$

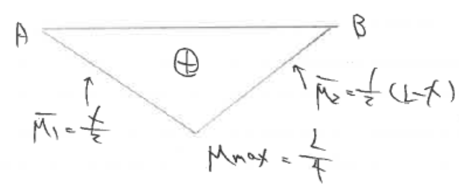


$\Sigma M: V_A x - 1 \cdot (x - \frac{L}{2}) - \bar{M}_2 = 0$

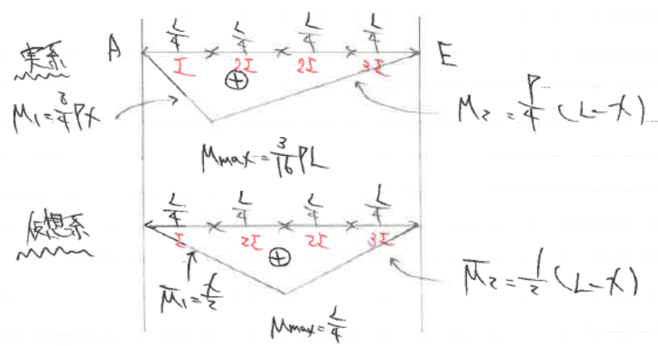
$\frac{1}{2}x - x + \frac{L}{2} = \bar{M}_2$

$\bar{M}_2 = \frac{1}{2}(L - x)$

< M図 >



実系と仮想系より、



問題4　解答

よって、

$$\Delta_C = \underbrace{\int_0^{\frac{L}{4}} \frac{\left(\frac{3}{4}Px\right)\left(\frac{x}{2}\right)}{EI}\,dx}_{①} + \underbrace{\int_{\frac{L}{4}}^{\frac{2}{4}L} \frac{\frac{P}{4}(L-x)\cdot\frac{x}{2}}{E\cdot 2I}\,dx}_{②} + \underbrace{\int_{\frac{2}{4}L}^{\frac{3}{4}L} \frac{\frac{P}{4}(L-x)\cdot\frac{1}{2}(L-x)}{E\cdot 2I}\,dx}_{③}$$

$$+ \underbrace{\int_{\frac{3}{4}L}^{L} \frac{\frac{P}{4}(L-x)\frac{1}{2}(L-x)}{E\cdot 3I}\,dx}_{④}$$

それぞれを計算

①、②、③、④より、

$$\Delta_C = \frac{PL^3}{512EI} + \frac{11}{3072EI}PL^3 + \frac{7}{3072EI}PL^3 + \frac{PL^3}{4608EI}$$

$$= \frac{36PL^3 + 66PL^3 + 42PL^3 + 4PL^3}{18432EI}$$

$$= \frac{148PL^3}{18432EI} = \frac{37PL^3}{4608EI}$$

$$\Delta_C = \frac{37PL^3}{4608EI}$$