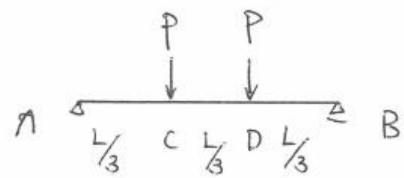
演習問題①)

① C点の鉛直方向のたわみを求めよ.



$\Sigma H: H_A = 0$

$\Sigma V: V_A + V_B - P - P = 0$

$\Sigma M_{(A)}: P \cdot \frac{L}{3} + P \cdot \frac{2}{3}L - V_B L = 0$

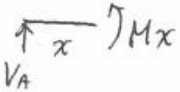
$V_B = P$

$V_A = P$

※曲げによるせん断ひずみエネルギーは無視するものとする

解答①) ◎実系について

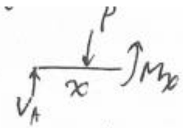
i) $0 \leq x < \frac{L}{3}$



$M_x - V_A x = 0$

$M_x = Px$

ii) $\frac{L}{3} \leq x < \frac{2}{3}L$

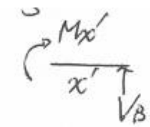


$M_x - V_A x - P(x - \frac{L}{3}) = 0$

$M_x = Px - Px + \frac{P}{3}L$

$M_x = \frac{PL}{3}$

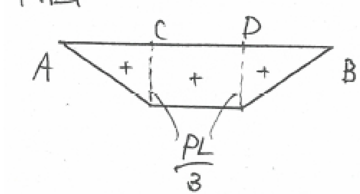
iii) $0 \leq x' \leq \frac{L}{3}$



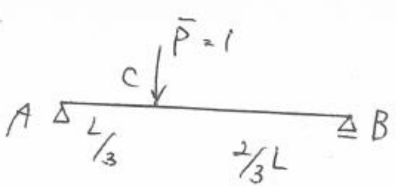
$M_{x'} - V_B x' = 0$

$M_{x'} = Px'$

M図



◎仮想系について(C点に鉛直方向の仮想仕事$\overline{P}$=1を作用)



$\Sigma H: H_A = 0$

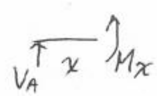
$\Sigma V: V_A + V_B - 1 = 0$

$\Sigma M_{(A)}: \bar{P} \times \frac{L}{3} - V_B L = 0$

$V_B = \frac{1}{3}$

$V_A = \frac{2}{3}$

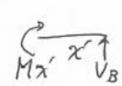
i) $0 \leq x < \frac{L}{3}$



$\bar{M}_x - V_A x = 0$

$\bar{M}_x = \frac{2}{3}x$

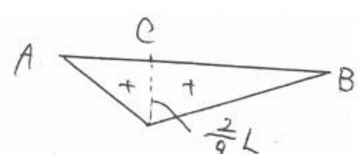
ii) $0 \leq x' \leq \frac{2}{3}L$



$\bar{M}_{x'} - V_B x' = 0$

$\bar{M}_{x'} = \frac{1}{3}x'$

$\bar{M}$図



$$\delta = \int_L \frac{M\bar{M}}{EI}dx$$

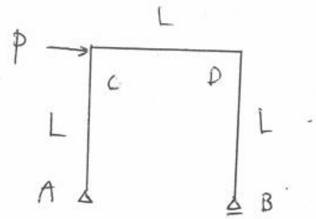
$$= \int_0^{L/3} \frac{(Px)(\frac{2}{3}x)}{EI}dx + \int_0^{L/3} \frac{(Px')(\frac{1}{3}x')}{EI}dx' + \int_{L/3}^{\frac{2}{3}L} \frac{(\frac{PL}{3})(\frac{1}{3}x')}{EI}dx'$$

$$= \left[\frac{2Px^3}{9EI}\right]_0^{L/3} + \left[\frac{Px'^3}{9EI}\right]_0^{L/3} + \left[\frac{PLx'^2}{18EI}\right]_{L/3}^{2/3}$$

$$= \underline{\frac{5PL^3}{162EI}}$$

演習問題②）

② C点の水平変位を求めよ．



$\Sigma H: H_A + P = 0,\ H_A = -P$

$\Sigma V: V_A + V_B = 0$

$\Sigma M_{(A)}: PL - V_B L = 0$

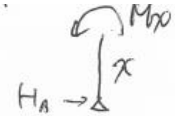
$V_B = P$

$V_A = -P$

※曲げによるせん断ひずみエネルギーは無視するものとする

解答②） ◎実系について

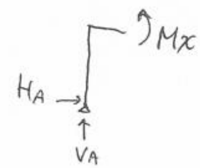
$A \leq x \leq C$



$M_x + H_A x = 0$

$M_x = -H_A x = Px$

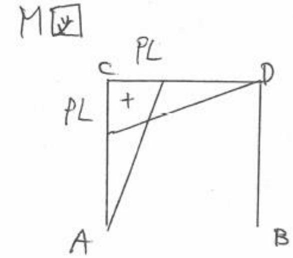
$C \leq x \leq D$



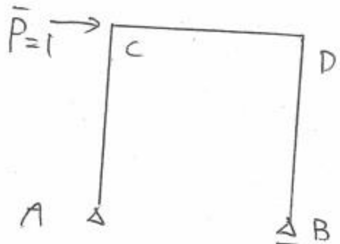
$M_x = V_A x - H_A L$

$= -Px + PL$

M図



◎仮想系について(C点に水平方向の仮想仕事$\overline{P}$=1を作用)



$\Sigma H: H_A = -1$

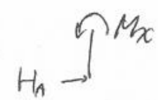
$\Sigma V: V_A + V_B = 0$

$\Sigma M_{(A)}: L - V_B L = 0$

$V_B = 1$

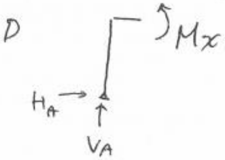
$V_A = -1$

$A \leq x \leq C$

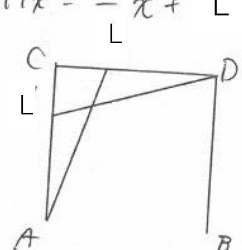


$M_x = x$

$C \leq x \leq D$



$M_x = -x + L$



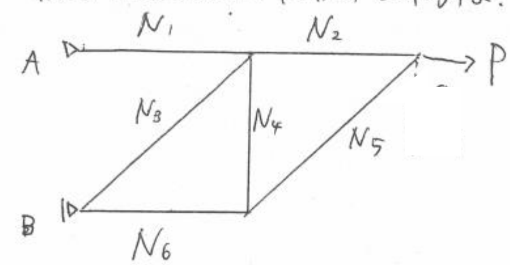
$$\delta = \int_L \frac{M\bar{M}}{EI}dx$$

$$= \int_0^L \frac{(Px)(x)}{EI}dx + \int_0^L \frac{(-Px+PL)(-x+L)}{EI}dx$$

$$= \frac{2PL^3}{3EI}$$

演習問題③）

③ C点の鉛直方向のたわみを求めよ.



$\Sigma V: V_A = 0$

$\Sigma H: H_A + H_B + P = 0$

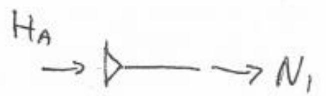
$\Sigma M_{(B)}: H_A L + PL = 0$

$H_A = -P$

$H_B = 0$

解答③） ◎<u>実系について</u>

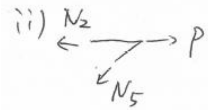
i)



$H_A + N_1 = 0$

$N_1 = -H_A = P$

ii)



$N_5 = 0$

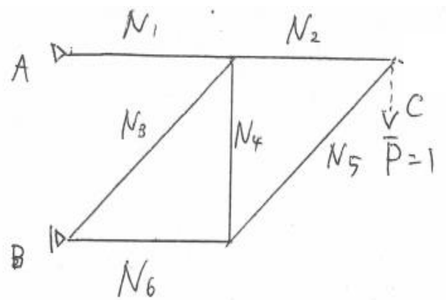
$N_2 = P$

$N_3 = 0$

$N_6 = 0$

$N_4 = 0$

◎<u>仮想系について(C点に鉛直方向の仮想仕事$\overline{P}$=1を作用)</u>



$\bar{P} = 1$のとき

$\Sigma V: V_A = 1$

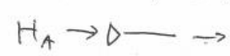
$\Sigma H: H_A + H_B = 0$

$M_{(B)}: H_A \times L + 1 \times 2L = 0$

$H_A = -2$

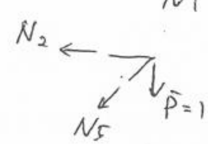
$H_B = 2$

必要なのは$\bar{N}_1$, $\bar{N}_2$だけなので



$H_A + \bar{N}_1 = 0$

$\bar{N}_1 = 2$



$N_2 + N_5 \times \frac{1}{\sqrt{2}} = 0$

$N_5 \times \frac{1}{\sqrt{2}} + 1 = 0$

$N_5 = -\sqrt{2}$

$N_2 = 1$

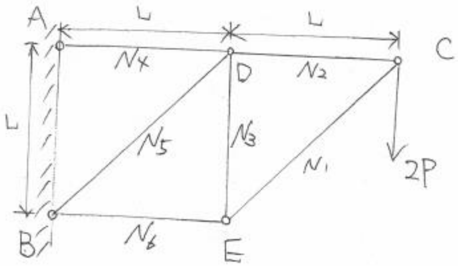
i	N_i	$\bar{N}_i$	L_i	$N_i\bar{N}_iL_i$
1	P	2	L	2PL
2	P	1	L	PL
3	0		$\sqrt{2}L$	0
4	0		L	0
5	0		$\sqrt{2}L$	0
6	0		L	0

$\delta = \Sigma \frac{N_i\bar{N}_iL_i}{EA}$

$= \frac{3PL}{EA}$

演習問題④)

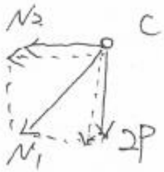
④ C点における鉛直下方変位を求めよ.



部材番号を上図の番号とする.
トラス部材の部材力を節点法により求める.

解答④)

1) C点でのつり合い



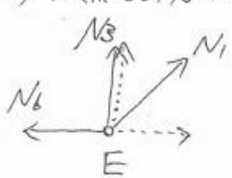
$\Sigma H: N_2 + \frac{1}{\sqrt{2}} N_1 = 0$

$\Sigma V: 2P + \frac{1}{\sqrt{2}} N_1 = 0$

$\therefore N_1 = -2\sqrt{2}P, \quad N_2 = 2P$

(N_1を鉛直方向,水平方向に分解).

2) E点での力のつり合いより



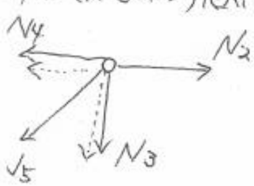
$\Sigma H: \frac{1}{\sqrt{2}} N_1 - N_6 = 0$

$\therefore N_6 = -2P$

$\Sigma V: N_3 + \frac{1}{\sqrt{2}} N_1 = 0$

$\therefore N_3 = 2P$

3) D点でのつり合いより



$\Sigma H: N_2 - N_4 - \frac{1}{\sqrt{2}} N_5 = 0$

$\Sigma V: N_3 + \frac{1}{\sqrt{2}} N_5 = 0$

$\therefore N_5 = -2\sqrt{2}P, \quad N_4 = 4P$

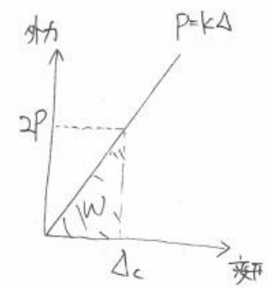
部材	L_i	N_i	N_i^2	$N_i^2 L$
1	$\sqrt{2}L$	$-2\sqrt{2}P$	$8P^2$	$8\sqrt{2}P^2L$
2	L	$2P$	$4P^2$	$4P^2L$
3	L	$2P$	$4P^2$	$4P^2L$
4	L	$4P$	$16P^2$	$16P^2L$
5	$\sqrt{2}L$	$-2\sqrt{2}P$	$8P^2$	$8\sqrt{2}P^2L$
6	L	$-2P$	$4P^2$	$4P^2L$

$\Sigma N_i^2 l = 4(7+4\sqrt{2})P^2L$

・内力エネルギー U

$$U = \frac{\Sigma N_i^2 l}{2EA} = \frac{4(7+4\sqrt{2})}{2EA}P^2L = \frac{2(7+4\sqrt{2})}{EA}P^2L$$

・外力エネルギー W



$$W = \frac{1}{2} \times 2P \times \Delta_c = P\Delta_c$$

仕事エネルギー的に評価す

$W = U$ より

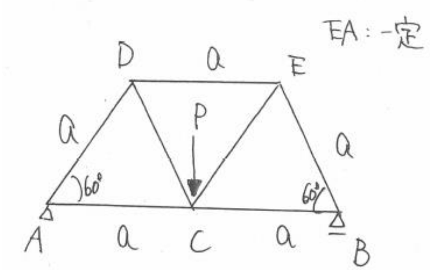
$$P\Delta_c = \frac{2(7+4\sqrt{2})}{EA}P^2L$$

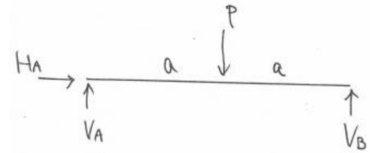
$$\therefore \Delta_c = \frac{2(7+4\sqrt{2})}{EA}PL$$

//

演習問題⑤)

⑤ C点のたわみを求めよ.





$\Sigma H = 0$: $H_A = 0$

$\Sigma V = 0$: $V_A + V_B - P = 0$

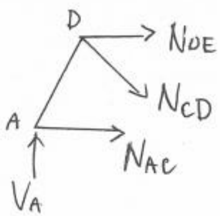
$\Sigma M_{(A)} = 0$: $Pa - V_B \cdot 2a = 0$

$$V_B = \frac{P}{2}$$

$$V_A = \frac{P}{2}$$

解答⑤)

断面法より



$\Sigma H = 0$: $N_{DE} + N_{CD} \times \cos 60° + N_{AC} = 0$

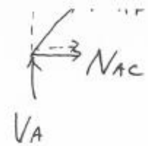
$\Sigma V = 0$: $V_A - N_{CD} \times \sin 60° = 0$

$\Sigma M_{(D)} = 0$: $V_A \cdot \frac{a}{2} - N_{AC} \times \cos 30° \times a = 0$

$$N_{CD} = \frac{P}{\sqrt{3}}$$

$$N_{AC} = \frac{P}{2\sqrt{3}}$$

$$N_{DE} = -\frac{P}{\sqrt{3}}$$



$$N_{AD} \cos 60° = -N_{AC}$$

$$N_{AD} = \frac{-P}{\sqrt{3}}$$

左右対称の構造,荷重のため

$$N_{AD} = N_{BE} = \frac{-P}{\sqrt{3}}$$

$$N_{AC} = N_{BC} = \frac{P}{2\sqrt{3}}$$

$$N_{CD} = N_{CE} = \frac{P}{\sqrt{3}}$$

$U = \Sigma \frac{N^2 L}{2EA}$ より

$$U = \frac{a}{2EA}\left\{\left(\frac{-P}{\sqrt{3}}\right)^2 + \left(\frac{-P}{\sqrt{3}}\right)^2 + \left(\frac{P}{2\sqrt{3}}\right)^2 + \left(\frac{P}{2\sqrt{3}}\right)^2 + \left(\frac{P}{\sqrt{3}}\right)^2 + \left(\frac{P}{\sqrt{3}}\right)^2 + \left(\frac{-P}{\sqrt{3}}\right)^2\right\}$$

$$= \frac{11P^2 a}{12EA}$$

$$W = \frac{1}{2} P\Delta$$

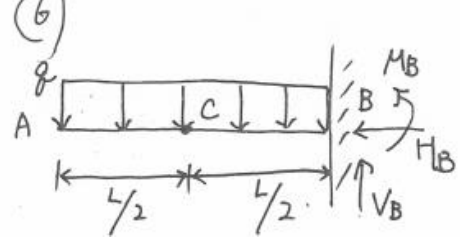
$W = U$ より

$$\frac{1}{2} P\Delta = \frac{11P^2 a}{12EA}$$

$$\underline{\Delta = \frac{11Pa}{6EA}}$$

演習問題⑥）

⑥ C点の鉛直たわみを求めよ.



$\Sigma H = H_B = 0$

$\Sigma V = V_B - qL = 0$

$\therefore V_B = qL$

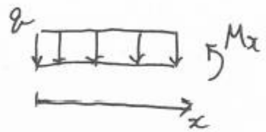
$\Sigma M_{(B)} = -M_B - \dfrac{qL^2}{2} = 0$

$\therefore M_B = -\dfrac{qL^2}{2}$

※曲げによるせん断ひずみエネルギーは無視するものとする

解答⑥）　◎実系について

i) $0 < x < L$



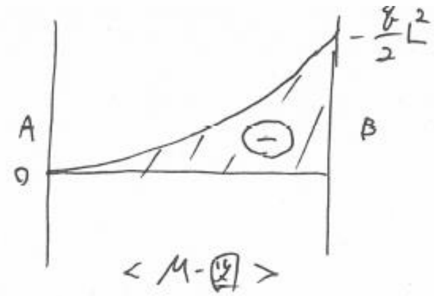
$\Sigma M_{(x)} = -M_x - qx \cdot \dfrac{x}{2} = 0$

$\therefore M_x = -\dfrac{q}{2}x^2$

$\dfrac{M\bar{M}}{EI}\,dx$ より

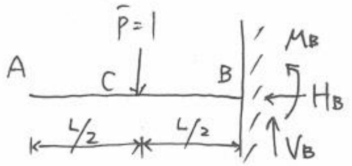
$\dfrac{(-\frac{q}{2}x^2)\cdot 0}{EI}\,dx + \int$

$0 + \dfrac{1}{EI}\left[\dfrac{q}{8}x^4 -\right.$



◎仮想系について(C点に鉛直方向の仮想仕事$\bar{P}$=1を作用)

(仮想系)

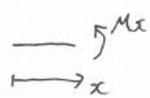


$\Sigma H = H_B = 0$

$\Sigma V = V_B - 1 = 0 \quad \therefore V_B = 1$

$\Sigma M_{(B)} = -M_B - 1 \cdot L/2 = 0 \quad \therefore M_B = -\dfrac{L}{2}$

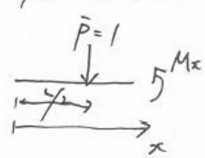
i) $0 < x < L/2$



$\Sigma M_{(x)} = -M_x = 0$

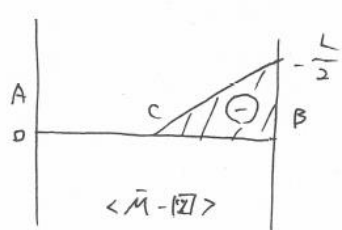
$\therefore M_x = 0$

ii) $L/2 < x < L$



$\Sigma M_{(x)} = -M_x - \bar{P}\cdot(x - \dfrac{L}{2})$

$\therefore M_x = -(x - \dfrac{L}{2})$



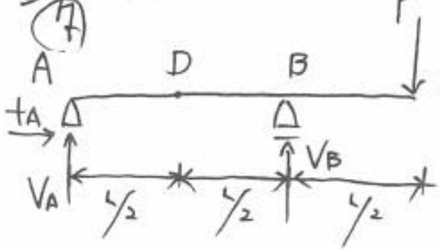
$V_c = \displaystyle\int_0^L \frac{M\bar{M}}{EI}\,dx$ より

$V_c = \displaystyle\int_0^{\frac{L}{2}} \frac{(-\frac{q}{2}x^2)\cdot 0}{EI}\,dx + \int_{L/2}^{L} \frac{-(-\frac{q}{2}x^2)\cdot(x-\frac{L}{2})}{EI}\,dx$

$= 0 + \dfrac{1}{EI}\left[\dfrac{q}{8}x^4 - \dfrac{qL}{12}x^3\right]_{L/2}^{L} = \underline{\dfrac{17qL^4}{384EI}}$

演習問題⑦）

(3) D点の鉛直方向のたわみを求めよ



・反力

$\Sigma H = H_A = 0$

$\Sigma V = V_A + V_B - P = 0$

$\Sigma M_{(A)} = P \cdot \frac{3}{2}L - V_B L = 0$

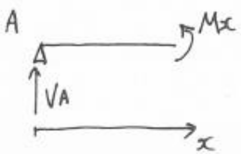
$\therefore V_B = \frac{3}{2}P, \quad V_A = -\frac{P}{2}$

※曲げによるせん断ひずみエネルギーは無視するものとする

解答⑦）　◎実系について

・モーメントを求める.

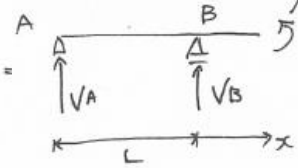
i) $0 < x < L$



$\Sigma M_{(x)} = -M_x + V_A x =$

$\therefore M_x = -\frac{P}{2}x$

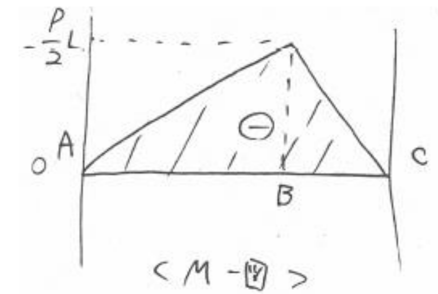
ii) $L < x < \frac{3}{2}L$



$\Sigma M_{(x)} = -M_x + V_A x + V_B(x - L) = 0$

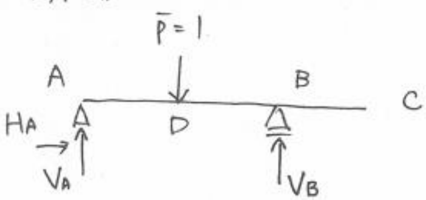
$\therefore M_x = -\frac{P}{2}x + \frac{3P}{2}(x - L)$

$= Px - \frac{3}{2}PL$



◎仮想系について(D点に鉛直方向の仮想仕事$\overline{P}$=1を作用)

(仮想系)



・反力

$\Sigma H = H_A = 0$

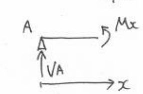
$\Sigma V = V_A + V_B - 1 = 0$

$\Sigma M_{(A)} = 1 \cdot \frac{L}{2} - V_B L = 0$

$\therefore V_B = \frac{1}{2}, \quad V_A = \frac{1}{2}$

モーメントを求める

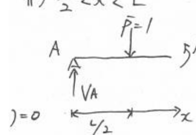
i) $0 < x < L/2$



$\Sigma M_{(x)} = -M_x + V_A x = 0$

$\therefore M_x = \frac{1}{2}x$

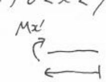
ii) $\frac{L}{2} < x < L$



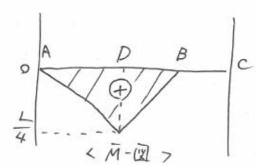
$\Sigma M_{(x)} = -M_x - 1 \cdot (x - L/2) + V_A x = 0$

$\therefore M_x = -\frac{1}{2}x + \frac{L}{2}$

iii) $0 < x' < L/2$　(右側から考える).



$\Sigma M_{(x')} = M_{x'} = 0$



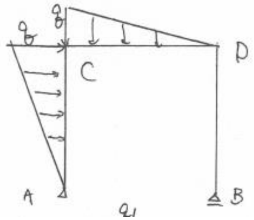
$\therefore V = \int_0 \frac{M \bar{M}}{EI} dx$ より

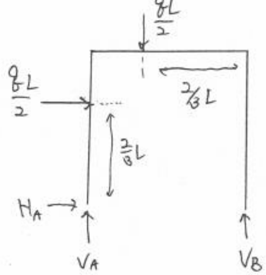
$V_D = \frac{1}{EI}\left\{ \int_0^{L/2} \left(-\frac{P}{2}x\right)\left(\frac{1}{2}x\right) dx + \int_{L/2}^{L} \left(-\frac{P}{2}x\right)\left(-\frac{1}{2}x + \frac{L}{2}\right) dx \right\}$

$= \frac{1}{EI}\left(\left[-\frac{P}{12}x^3\right]_0^{L/2} + \left[\frac{P}{12}x^3 - \frac{PL}{8}x^2\right]_{L/2}^{L} \right)$

$= -\frac{PL^3}{32EI}$

演習問題⑧） (8) C点の水平変位を求めよ





EI:一定

$\Sigma H: H_A + \frac{qL}{2} = 0$

$\Sigma V: V_A + V_B - \frac{qL}{2} = 0$

$\Sigma M_{(A)}: \frac{qL}{2} \times \frac{2}{3}L + \frac{qL}{2} \times \frac{1}{3}L - V_B L = 0$

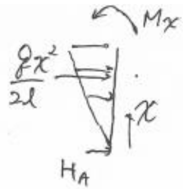
$H_A = \frac{qL}{2}$

$V_A = 0$

$V_B = \frac{qL}{2}$

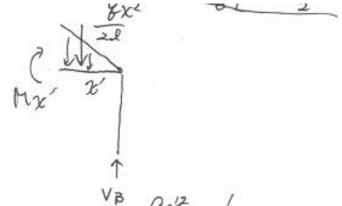
※曲げによるせん断ひずみエネルギーは無視するものとする

解答⑧） ◎実系について



$M_x + H_A x + \frac{qx^2}{2l} \cdot \frac{x}{3} = 0$

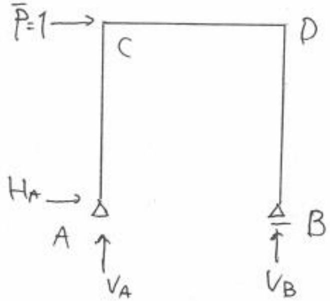
$M_x = -\frac{qx^3}{6L} + \frac{qL}{2}x$



$M_{x'} + \frac{qx'^2}{2L} \times \frac{x'}{3} - V_B x' = 0$

$M_{x'} = \frac{qL}{2}x' - \frac{qx'^3}{6L}$

◎仮想系について(C点に水平方向の仮想仕事$\overline{P}$=1を作用)



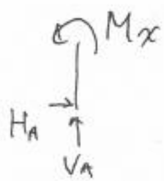
$\Sigma H: H_A + 1 = 0 \quad H_A = -1$

$\Sigma V: V_A + V_B = 0$

$\Sigma M_{(A)}: 1 \cdot L - V_B \cdot L = 0$

$V_B = 1$

$V_A = -1$



$M_x + H_A x = 0$

$M_x = x$

$M_{x'} - V_B x = 0$

$M_{x'} = x$

$$\delta = \int \frac{\bar{M}M}{EI}dx = \int_0^L \frac{\left(-\frac{qx^3}{6L} + \frac{qL}{2}x\right)\cdot x}{EI} + \int_0^L \frac{\left(\frac{qL}{2}x' - \frac{qx'^3}{6L}\right)\cdot x'}{EI}$$

$$= \left[-\frac{qx^5}{30L} + \frac{qL}{6}x^3\right]_0^L + \left[\frac{qx'^3}{6} - \frac{qx^5}{30L}\right]_0^L$$

$$= \frac{4qL^4}{15EI}$$