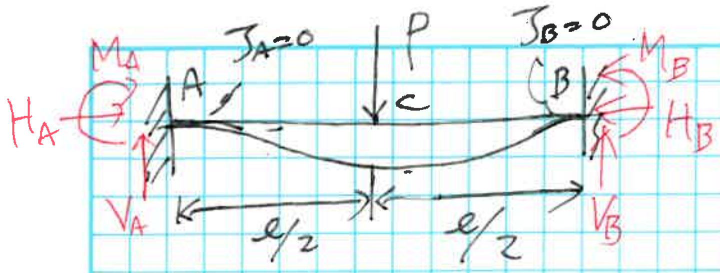
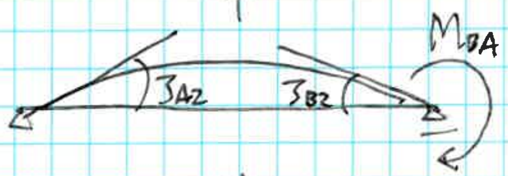
◎ たわみ角式（端モーメント式）の誘導



⇓ 力学的に等価：微小変形、弾性域

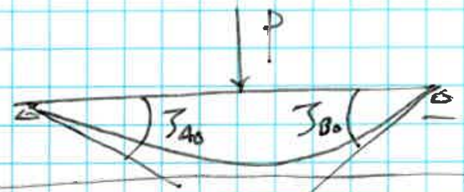
端モーメント $M_{AB}=M_A$ ＜1系＞

＋



端モーメント $M_{BA}=-M_B$ ＜2系＞

＋



荷重項 ＜0系＞

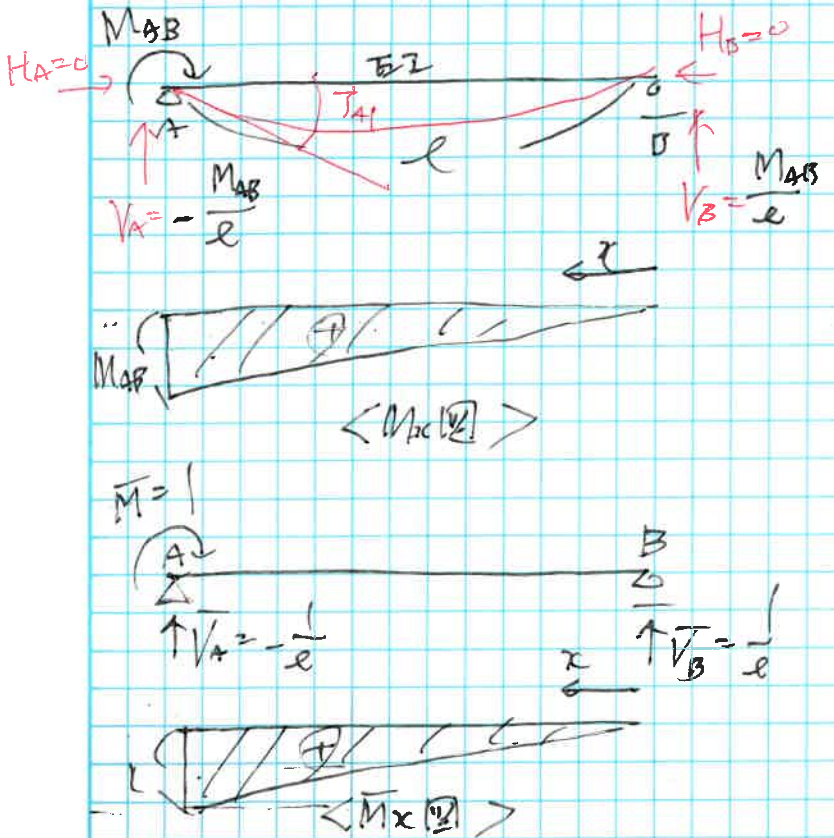
ここで、部材角 $R=0$（A点、B点の移動なし）

$\therefore\ \theta_A=\tau_A+R=\tau_A\ ,\quad \theta_B=\tau_B+R=\tau_B$

よって 以下のことが成立する。

$$\left\{\begin{array}{l}\tau_A=\tau_{A1}+\tau_{A2}+\tau_{A0}=0\\ \tau_B=\tau_{B1}+\tau_{B2}+\tau_{B0}=0\end{array}\right\}\ \text{重ね合わせの原理}$$

○ θ_{A1} を仮想仕事法で求める.



$$EI\theta_{A1}=\int_0^l M_x\overline{M}_x dx$$

$$M_x=\frac{M_{AB}}{l}x \quad \overline{M}_x=\frac{1}{l}x$$

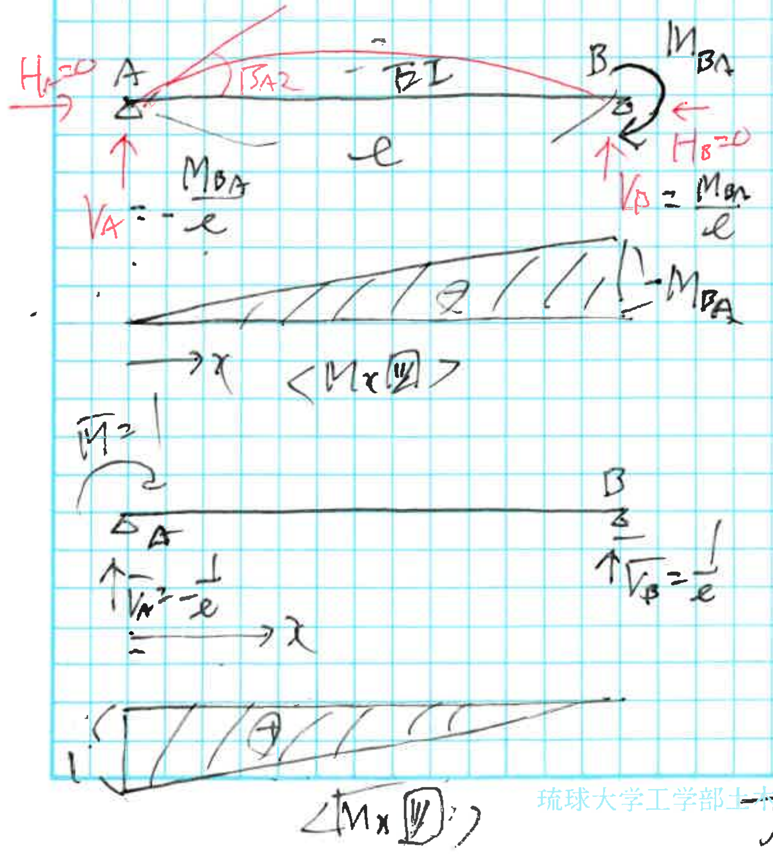
$$EI\theta_{A1}=\int_0^l\frac{M_{AB}}{l^2}x^2dx$$

$$=\frac{M_{AB}}{l^2}\left[\frac{x^3}{3}\right]_0^l$$

$$=\frac{M_{AB}l^3}{3l^2}=\frac{M_{AB}l}{3}$$

$$\theta_{A1}=\frac{M_{AB}l}{3EI} \rightarrow ①$$

○ θ_{A2} を仮想仕事法で求める.



$$EI\theta_{A2}=\int_0^l M_x\overline{M}_x dx$$

$$\begin{cases} M_x=-\frac{M_{BA}}{l}x \\ \overline{M}_x=1-\frac{1}{l}x \end{cases}$$

$$EI\theta_{A2}=\int_0^l\left(-\frac{M_{BA}}{l}x\right)\left(1-\frac{1}{l}x\right)dx$$

$$=\int_0^l\left(-\frac{M_{BA}}{l}x+\frac{M_{BA}}{l^2}x^2\right)dx$$

$$=\left[-\frac{M_{BA}}{2l}x^2+\frac{M_{BA}}{3l^2}x^3\right]_0^l$$

$$=-\frac{M_{BA}}{2l}\times l^2+\frac{M_{BA}}{3l^2}l^3$$

$$=-\frac{M_{BA}}{2}l+\frac{M_{BA}}{3}l$$

$$\theta_{A2}=-\frac{M_{BA}l}{6EI} \rightarrow ②$$

同様に B点について θ_{B1} と θ_{B2} を仮想仕事法で求めると

$$\theta_{B1} = -\frac{M_{AB}\,l}{6EI}, \quad \theta_{B2} = \frac{M_{BA}\,l}{3EI}$$

よって、

$$\theta_A = \theta_{A1} + \theta_{A2} + \theta_{A0} = \frac{M_{AB}\,l}{3EI} - \frac{M_{BA}\,l}{6EI} + \theta_{A0} \quad \text{—③}$$

$$\theta_B = \theta_{B1} + \theta_{B2} + \theta_{B0} = -\frac{M_{AB}\,l}{6EI} + \frac{M_{BA}\,l}{3EI} + \theta_{B0} \quad \text{—④}$$

上式を端モーメントについて整理すると

$$\frac{M_{AB}\,l}{3EI} = \theta_A - \theta_{A0} + \frac{M_{BA}\,l}{6EI}$$

$$M_{AB} = \frac{3EI}{l}(\theta_A - \theta_{A0}) + \frac{M_{BA}}{2} \quad \text{-----⑤}$$

式⑤を式④に代入すると

$$\theta_B = -\frac{l}{6EI} \times \left\{ \frac{3EI}{l}(\theta_A - \theta_{A0}) + \frac{1}{2}M_{BA} \right\} + \frac{M_{BA}}{3EI}l + \theta_{B0}$$

$$= -\frac{1}{2}(\theta_A - \theta_{A0}) - \frac{l}{12EI}M_{BA} + \frac{l}{3EI}M_{BA} + \theta_{B0}$$

$$\left(\frac{l}{12EI} - \frac{l}{3EI}\right)M_{BA} = -\frac{1}{2}\theta_A + \frac{1}{2}\theta_{A0} - \theta_B + \theta_{B0}$$

$$-\frac{l}{4EI}M_{BA} = -\frac{1}{2}\theta_A - \theta_B + \frac{1}{2}\theta_{A0} + \theta_{B0}$$

$$M_{BA} = \frac{4EI}{l} \times \left(\frac{1}{2}\theta_A + \theta_B - \frac{1}{2}\theta_{A0} - \theta_{B0}\right)$$

$$= \frac{2EI}{l}(\theta_A + 2\theta_B) - \frac{2EI}{l}(\theta_{A0} + 2\theta_{B0}) \quad \text{—⑥}$$

$\frac{1-4}{12EI}$

$-\frac{3}{12}$

⑥を⑤へ代入して

$$M_{AB} = \frac{2EI}{l}(2\theta_A + \theta_B) - \frac{2EI}{l}(2\theta_{A0} + \theta_{B0}) \quad \text{—⑦}$$

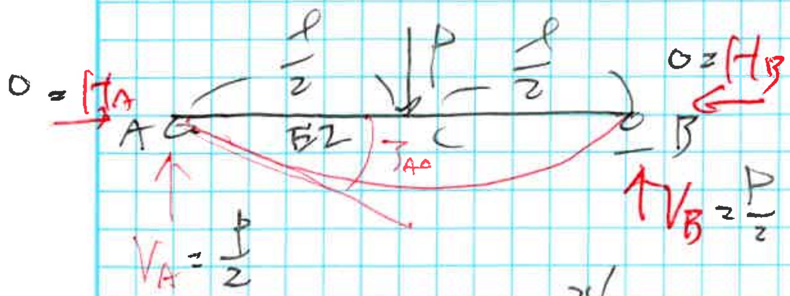
$$\begin{cases} C_{AB} = -\dfrac{2EI}{l}(2\tau_{A0} + \tau_{B0}) \\ C_{BA} = -\dfrac{2EI}{l}(\tau_{A0} + 2\tau_{B0}) \end{cases}$$ とおくと —⑧

また $\theta_A = \tau_A + R$, $\theta_B = \tau_B + R$ を用いると —⑨

$$\boxed{\begin{cases} M_{AB} = \dfrac{2EI}{l}(2\theta_A + \theta_B - 3R) + C_{AB} \\ M_{BA} = \dfrac{2EI}{l}(\theta_A + 2\theta_B - 3R) + C_{BA} \end{cases}}$$ —⑩

◎ 荷重項 C_{AB}, C_{BA} を求める. (表8.1)

◎ τ_{A0} を仮想仕事法で求める



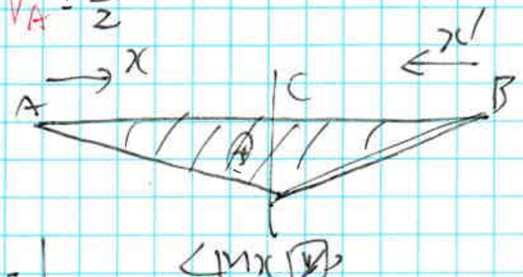
$$EI\tau_{A0} = \int_A^C M_x\bar{M}_x dx + \int_B^C M_x\bar{M}_x dx'$$

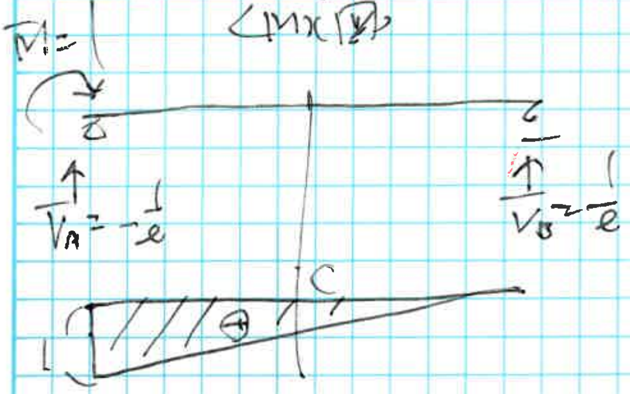
A～C

$$M_x = \frac{P}{2}x, \quad \bar{M}_x = 1-\frac{1}{l}x$$

B～C

$$M_x = \frac{P}{2}x', \quad \bar{M}_x = \frac{1}{l}x'$$





$$EI\tau_{A0} = \int_0^{\frac{l}{2}} \frac{P}{2}x\left(1-\frac{1}{l}x\right)dx + \int_0^{\frac{l}{2}} \frac{P}{2l}x'^2 dx'$$

$$= \int_0^{\frac{l}{2}}\left(\frac{P}{2}x - \frac{P}{2l}x^2\right)dx + \int_0^{\frac{l}{2}} 〃$$

$$= \left[\frac{P}{4}x^2 - \frac{P}{6l}x^3\right]_0^{\frac{l}{2}} + \left[\frac{P}{6l}x'^3\right]_0^{\frac{l}{2}}$$

$$= \frac{Pl^2}{16} - \frac{Pl^3}{48l} + \frac{Pl^3}{48l}$$

$$= \frac{Pl^2}{16}$$

$$\tau_{A0} = \frac{Pl^2}{16EI}$$

同様に $\tau_{B0} = -\frac{Pl^2}{16EI}$

$$C_{AB} = -\frac{2EI}{l}(2\tau_{A0} + \tau_{B0})$$

$$= -\frac{2EI}{l}\times\left(\frac{2Pl^2}{16EI} - \frac{Pl^2}{16EI}\right)$$

$$= -\frac{Pl^2}{8} \quad //$$

∴ $C_{BA} = \frac{Pl^2}{8} = -C_{AB}$